Exact soliton solutions to coupled nonlinear Schrödinger equations with higher-order effects

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We derive the exact bright and dark soliton solutions of a set of coupled nonlinear Schrödinger equations with higher-order linear and nonlinear dispersion terms included, for a specific set of parameters by using the Hirota bilinear approach, so as to get some ideas about the role of these terms for coupled soliton dynamics. Further, by considering the additional nonlinear effects resulting from the delayed nonlinear response, we have noted corresponding changes in the above soliton solutions. [S1063-651X(96)01509-7]

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I. INTRODUCTION

The dynamics of a nonlinear short-optical pulse envelope in a fiber is described by

$$iq_z-\frac{k''}{2}q_{tt}+\beta|q|^2q-\frac{ik'''}{6}q_{ttt}+i\gamma(|q|^2q)_t=0, \tag{1}$$

which becomes the nonlinear Schrödinger (NLS) equation when terms proportional to k''' and γ are negligible [1–3]. However, in some regions, the role of k''' and γ becomes important. In particular, to describe the effects of pulse broadening in the frequency region where k'' is close to zero, one needs to take k''' to be non-negligible [4,5]. The last term proportional to γ becomes important for short-pulse propagation over long distances [6,7]. In Eq. (1) q represents the complex envelope amplitude, t and z are the time and distance along the direction of propagation, k'' is the second derivative of the axial wave number k with respect to the angular frequency ω at the central frequency ω_0 and describes group velocity dispersion (GVD), $k'''=\partial^3k/\partial\omega^3$ at ω_0 describes third order dispersion, $\beta=n_2\omega_0/cA_{\rm eff}$ is the effective nonlinear coefficient where n_2 is the Kerr coefficient, and c is the velocity of light, $A_{\rm eff}$ is the effective core area of the fiber and $\gamma=2\beta/\omega_0$ describes nonlinear dispersion.

There are several ways to generalize the system (1) to a set of coupled equations depending on the physical situation that is being modeled [1–3,8,9]. A fairly general form of coupled equations can be written from (1) as

$$iq_{1z}-\frac{k''}{2}q_{1tt}+\beta[(a|q_1|^2+b|q_2|^2)q_1]-\frac{ik'''}{6}q_{1ttt}+i\gamma[(a|q_1|^2+b|q_2|^2)q_1]_t=0, \tag{2a}$$

$$iq_{2z}-\frac{k''}{2}q_{2tt}+\beta[(b|q_1|^2+a|q_2|^2)q_2]-\frac{ik'''}{6}q_{2ttt}+i\gamma[(b|q_1|^2+a|q_2|^2)q_2]_t=0, \tag{2b}$$

where q_1 and q_2 represent the complex envelope amplitudes and a and b are numerical factors that depend on the physical situation. For $k'''=0$ the system (2) reduces to the coupled hybrid nonlinear Schrödinger equations and it has been recently derived by Hisakado, Iizuka, and Wadati [10] from the Maxwell equations in order to investigate the effects of birefringence on pulse propagation in the femtosecond range. They have also given an inverse scattering formulation for the system (2) with $a=b$ and $k'''=0$ [11]. Further, it is interesting to note that the system (2) also becomes the well-known integrable model proposed by Manakov [12] for the parametric choices $a=b$ and $k'''=\gamma=0$ and its bright and dark soliton solutions have been well studied recently [13,14] to explain many physical phenomena such as the birefringence property, soliton trapping, and daughter wave (''shadow'') formation in optical fibers. Recently in Ref. [15], we have obtained exact bright and dark N-soliton solutions for a system similar to (2), but which does not include the term proportional to γ in (2) and takes into account linear cross coupling systematically. However, as discussed earlier, under some physical situations, the role of k''' and γ becomes important and hence studies of (2) are quite important. By considering these facts, even though in general the system (2) may not be integrable, in this paper, using the Hirota technique [16,17], we obtain exact bright and dark soliton solutions of (2), under the conditions $a=b$ and $k''\gamma=\beta k'''$, so as to get some ideas of the role of k''' and γ in coupled soliton dynamics. Further, changes in these soliton solutions are noted when the additional nonlinear effects due to the delayed nonlinear response is considered in (2).

The plan of the paper is as follows. In Sec. II, we rewrite the system (2) in a Hirota bilinear form after using a suitable transformation and parametric condition. Sections III and IV are devoted to the derivation of exact bright and dark soliton solutions by using the Hirota technique and also to a discussion of their features. The effect of an additional higher-order nonlinear term in (2) is focused upon in Sec. V. Section VI is reserved as a conclusion.

II. HIROTA BILINEARIZATION

In order to construct soliton solutions of the system (2), it is rather convenient to introduce the transformation of variables

$$q_1(Z,T)=\rho_1(Z,T)\exp\{i[(k''/k''')T-(k''^3/6k'''^2)Z]\}, \tag{3a}$$

$$q_2(Z,T)=\rho_2(Z,T)\exp\{i[(k''/k''')T-(k''^3/6k'''^2)Z]\}, \tag{3b}$$

where the new variables Z and T are defined as

$$Z=z, \quad T=t+\frac{k''^2}{2k'''}z. \tag{4}$$

Using (3) in (2), we obtain, under the specific condition $k''\gamma=\beta k'''$ the following form of coupled envelope equations corresponding to the system (2):

$$\rho_{1Z}-\frac{k'''}{6}\rho_{1TTT}+\gamma[(a|\rho_1|^2+b|\rho_2|^2)\rho_1]_T=0, \tag{5a}$$

$$\rho_{2Z}-\frac{k'''}{6}\rho_{2TTT}+\gamma[(b|\rho_1|^2+a|\rho_2|^2)\rho_2]_T=0, \tag{5b}$$

where ρ_1 and ρ_2 are complex functions of Z and T. The reason for considering this specific choice here is that only for this case are we able to obtain suitable bilinear equations giving rise to N-soliton solutions of the system (2). For the case $\rho_2=0$, Eq. (5) reduces to the well-known complex modified Korteweg–de Vries equation, which arises naturally in the study of anharmonic lattices [18]. This shows that under appropriate conditions the optical solitons in fibers may have the same features as those of solitons in anharmonic lattices. It is also interesting to note that the system (5) can be obtained directly from (2) for the parametric choice $k''=\beta=0$ instead of using (3) in (2). Therefore by solving (5) one can obtain some ideas of the role of k''' and γ.

Next we wish to apply the following form of Hirota bilinear transformation to (5) in order to construct the bright and dark soliton solutions,

$$\rho_1=\frac{g}{f}, \quad \rho_2=\frac{h}{f}, \tag{6}$$

where $g(Z,T)$, $h(Z,T)$ are complex functions and $f(Z,T)$ is a real function.

Using (6) and the Hirota bilinear operators

$$D_T^m D_Z^n g(Z,T)\cdot f(Z,T)=\left(\frac{\partial}{\partial T}-\frac{\partial}{\partial T'}\right)^m\left(\frac{\partial}{\partial Z}-\frac{\partial}{\partial Z'}\right)^n g(Z,T)f(Z',T')\bigg|_{Z=Z',T=T'}, \tag{7}$$

where the centered dot stands for ordered multiplication by the preceding operators. Equations (5) can be rewritten as

$$f^2\left[\left(D_Z-\frac{k'''}{6}D_T^3\right)g\cdot f\right]+\left[\frac{k'''}{6}D_T^2 f\cdot f+\gamma(agg^*+bhh^*)\right][3D_T g\cdot f]+\gamma f[agD_T g^*\cdot g+bhD_T h^*\cdot g+bh^*D_T h\cdot g]=0, \tag{8a}$$

$$f^2\left[\left(D_Z-\frac{k'''}{6}D_T^3\right)h\cdot f\right]+\left[\frac{k'''}{6}D_T^2 f\cdot f+\gamma(bgg^*+ahh^*)\right][3D_T h\cdot f]+\gamma f[ahD_T h^*\cdot h+bgD_T g^*\cdot h+bg^*D_T g\cdot h]=0. \tag{8b}$$

III. EXACT BRIGHT SOLITON SOLUTIONS

For finding the exact bright N-soliton solutions Eqs. (8) can be decoupled under the condition $a=b$ as

$$\mathcal{A}_1 g\cdot f=0, \quad \mathcal{A}_1 h\cdot f=0, \quad \mathcal{A}_2 f\cdot f=-\gamma(gg^*+hh^*),$$

$$\mathcal{A}_3 g\cdot h=\mathcal{A}_3 g^*\cdot h=\mathcal{A}_3 g\cdot h^*=\mathcal{A}_3 g\cdot g^*=\mathcal{A}_3 h\cdot h^*=0, \tag{9a}$$

where the bilinear operators $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ are defined as

$$\mathcal{A}_1=\left(D_Z-\frac{k'''}{6}D_T^3\right), \quad \mathcal{A}_2=\frac{k'''}{6}D_T^2, \quad \mathcal{A}_3=D_T. \tag{9b}$$

Hereafter, we take $a=b=1$, without loss of generality. It can be noted that the last five equations in (9a) actually correspond to two independent equations, say $\mathcal{A}_3 g\cdot h=\mathcal{A}_3 g^*\cdot h=0$, from which the remaining three follow. Next we proceed in the standard way [16,17] to construct soliton solutions.

A. One-soliton solution

In order to find the one-soliton solution, we assume

$$g=\chi g_1, \quad h=\chi h_1, \quad f=1+\chi^2 f_2, \tag{10}$$

where χ is an arbitrary parameter. Substituting (10) into (9) and then collecting the terms with like powers of χ, we obtain

$$\mathcal{A}_1 g_1\cdot 1=0, \quad \mathcal{A}_1 h_1\cdot 1=0 \quad (\chi), \tag{11}$$

$$\mathcal{A}_2(1\cdot f_2+f_2\cdot 1)+\gamma(g_1g_1^*+h_1h_1^*)=0,$$

$$\mathcal{A}_3 g_1\cdot h_1=\mathcal{A}_3 g_1^*\cdot h_1=\mathcal{A}_3 g_1\cdot h_1^*=\mathcal{A}_3 g_1\cdot g_1^*=\mathcal{A}_3 h_1\cdot h_1^*$$
$$=0 \quad (\chi^2), \tag{12}$$

$$\mathcal{A}_1 g_1\cdot f_2=0, \quad \mathcal{A}_1 h_1\cdot f_2=0 \quad (\chi^3), \tag{13}$$

and

$$\mathcal{A}_2 f_2\cdot f_2=0 \quad (\chi^4). \tag{14}$$

One can easily check that the solution, which is consistent with the system (11)–(14), is

$$g_1=\exp(\eta_1+\eta_0), \quad h_1=\exp(\eta_1+\epsilon_0),$$

$$f_2=\frac{c_1}{P_1^2}\exp(2\eta_1), \tag{15}$$

where

$$\eta_1=P_1\left(T+\frac{k'''}{6}P_1^2 Z\right),$$

$$c_1=\frac{-3\gamma[\exp(\eta_0+\eta_0^*)+\exp(\epsilon_0+\epsilon_0^*)]}{4k'''}, \tag{16}$$

and in which η_0 and ϵ_0 are complex constants in general and all other parameters are real. Using (15) in (10) and then in (6), after absorbing χ, the bright one-soliton solution can be easily worked out to be

$$\rho_1=P_1\epsilon_1 \operatorname{sech}\left[P_1\left(T+\frac{k'''}{6}P_1^2 Z\right)+\delta_0\right], \tag{17a}$$

$$\rho_2=P_1\epsilon_2 \operatorname{sech}\left[P_1\left(T+\frac{k'''}{6}P_1^2 Z\right)+\delta_0\right], \tag{17b}$$

where

$$\epsilon_1=\left[\frac{\exp(2\eta_0)}{4c_1}\right]^{1/2}, \quad \epsilon_2=\left[\frac{\exp(2\epsilon_0)}{4c_1}\right]^{1/2}, \tag{18}$$

$$\delta_0=\tfrac{1}{2}\ln(c_1/P_1^2), \tag{19}$$

and from (16) and (18), we also have

$$|\epsilon_1|^2+|\epsilon_2|^2=\frac{-k'''}{3\gamma}, \tag{20}$$

It is obvious from (19) and (20) and the condition $k''\gamma=\beta k'''$ that k'' and β must have opposite signs, which naturally corresponds to the anomalous GVD region [19]. The bright one-soliton solution (17) appearing in the anomalous GVD region shows that the velocity of the soliton is proportional to the third-order dispersion and to the square of its pulse width. The explicit expression for q_1 and q_2 can be obtained by substituting (17) into the transformation (3). We can also verify that if we take $\epsilon_2=0$ and ρ_1 as a real function of Z and T, our result reduces to the case of the single extended NLS equation [20]. Further it is obvious from the resultant expressions for q_1, q_2 and (20) that the intensity of q_1, which is proportional to the ratio k'''/γ in the absence of q_2, is distributed among q_1 and q_2 when both are present.

B. Two-soliton solutions

Next in order to find the two-soliton solutions, we can assume

$$g=\chi g_1+\chi^3 g_3, \quad h=\chi h_1+\chi^3 h_3, \quad f=1+\chi^2 f_2+\chi^4 f_4, \tag{21}$$

and proceed as before to obtain

$$g_1=\exp(\eta_1+\eta_0)+\exp(\eta_2+\eta_0),$$

$$h_1=\exp(\eta_1+\epsilon_0)+\exp(\eta_2+\epsilon_0),$$

$$f_2=c_1\left[\frac{\exp(2\eta_1)}{P_1^2}+\frac{8\exp(\eta_1+\eta_2)}{(P_1+P_2)^2}+\frac{\exp(2\eta_2)}{P_2^2}\right],$$

$$g_3=\frac{c_1(P_1-P_2)^2}{(P_1+P_2)^2}\left[\frac{\exp(2\eta_1+\eta_2+\eta_0)}{P_1^2}+\frac{\exp(\eta_1+2\eta_2+\eta_0)}{P_2^2}\right],$$

$$h_3=\frac{c_1(P_1-P_2)^2}{(P_1+P_2)^2}\left[\frac{\exp(2\eta_1+\eta_2+\epsilon_0)}{P_1^2}+\frac{\exp(\eta_1+2\eta_2+\eta_0)}{P_2^2}\right],$$

$$f_4=\left[\frac{c_1^2(P_1-P_2)^4}{P_1^2P_2^2(P_1+P_2)^4}\right]\exp(2\eta_1+2\eta_2), \tag{22}$$

where

$$\eta_j=P_j\left(T+\frac{k'''}{6}P_j^2 Z\right), \quad j=1,2. \tag{23}$$

Here P_1 and P_2 are real parameters. Using (22) in (21) and then in (6), we obtain the explicit form of the two-soliton solutions as

$$\rho_1=\frac{2\epsilon_1(P_1+P_2)[P_2\cosh(\eta_1+\delta_2)+P_1\cosh(\eta_2+\delta_3)]}{|P_1-P_2|\{\cosh(\eta_1+\eta_2+\delta_4)+[(P_1+P_2)^2/(P_1-P_2)^2]\cosh(\eta_1-\eta_2+\delta_0-\delta_1)-c_2\}}, \tag{24a}$$

$$\rho_2=\frac{2\epsilon_2(P_1+P_2)[P_2\cosh(\eta_1+\delta_2)+P_1\cosh(\eta_2+\delta_3)]}{|P_1-P_2|\{\cosh(\eta_1+\eta_2+\delta_4)+[(P_1+P_2)^2/(P_1-P_2)^2]\cosh(\eta_1-\eta_2+\delta_0-\delta_1)-c_2\}}, \tag{24b}$$

where

$$\delta_1=\tfrac{1}{2}\ \ln(c_1/P_2^2),\quad \delta_{2(g)}=\tfrac{1}{2}\ \ln\left[\frac{c_1(P_1-P_2)^2}{P_{1(2)}^2(P_1+P_2)^2}\right],$$

$$\delta_4=\tfrac{1}{2}\ \ln\left[\frac{c_1^2(P_1-P_2)^4}{P_1^2P_2^2(P_1+P_2)^4}\right],\quad c_2=\frac{8P_1P_2}{(P_1-P_2)^2},\quad (24c)$$

and where ϵ_1 and ϵ_2 are defined as in (18).

In order to appreciate that the above solution is indeed the two-soliton solution, we rewrite (24) in terms of the new variable $\xi_1=T+(k'''/6)P_1^2Z$ and we allow the variable Z [for a fixed sign of $(P_2^2-P_1^2)$] to go to $\pm\infty$. Now we obtain in this limit

$$\rho_1\cong P_1\epsilon_1\ \mathrm{sech}\left[P_1\left(T+\frac{k'''}{6}\ P_1^2Z\right)\mp\delta'\right],\qquad (25a)$$

$$\rho_2\cong P_1\epsilon_2\ \mathrm{sech}\left[P_1\left(T+\frac{k'''}{6}\ P_1^2Z\right)\mp\delta'\right],\qquad (25b)$$

where the phase factor

$$\delta'=\tfrac{1}{2}\ \ln\left[\frac{(P_1+P_2)^2}{(P_1-P_2)^2}\right].\qquad (25c)$$

It is obvious from the one-soliton solution (25) that except for the phase shift of magnitude $2\delta'$, the form of the solutions in the limits $+\infty$ and $-\infty$ are the same. The presence of another set of one-soliton solutions in (24) with a similar behavior can also be shown by allowing the variable Z to go to $\pm\infty$ after introducing the new variable $\xi_2=T+(k'''/6)P_2^2Z$. Now we have as $Z\rightarrow\pm\infty$,

$$\rho_1\cong P_2\epsilon_1\ \mathrm{sech}\left[P_2\left(T+\frac{k'''}{6}\ P_2^2Z\right)\pm\delta'\right],\qquad (26a)$$

$$\rho_2\cong P_2\epsilon_2\ \mathrm{sech}\left[P_2\left(T+\frac{k'''}{6}\ P_2^2Z\right)\pm\delta'\right].\qquad (26b)$$

It is thus clear that the two solitons undergo elastic collisions. Finally the expression for q_1 and q_2 can be obtained explicitly after using (24) in the transformation (3).

C. Multisoliton solutions

Proceeding further one can generalize the expression for g, h, and f corresponding to the N-soliton solutions as

$$g(Z,T)=\sum_{\mu=0,1} M_1(\mu)\exp\left(\sum_{j=1}^{2N}\ \mu_j\eta_j+\sum_{1\leqslant i<j}^{2N}\ \mu_i\mu_j\phi_{ij}\right),\qquad (27a)$$

$$h(Z,T)=\sum_{\mu=0,1} M_2(\mu)\exp\left(\sum_{j=1}^{2N}\ \mu_j\eta_j+\sum_{1\leqslant i<j}^{2N}\ \mu_i\mu_j\phi_{ij}\right),\qquad (27b)$$

$$f(Z,T)=\sum_{\mu=0,1} M_3(\mu)\exp\left(\sum_{j=1}^{2N}\ \mu_j\eta_j+\sum_{1\leqslant i<j}^{2N}\ \mu_i\mu_j\phi_{ij}\right),\qquad (27c)$$

where

$$\eta_j=P_j\left(T+\frac{k'''}{6}\ P_j^2Z\right),\quad j=1,2,\dots,N,$$

$$\eta_{j+N}=\eta_j,\quad P_{j+N}=P_j\quad \text{for}\ \ j=1,2,\dots,N,$$

$$\exp(\phi_{ij})=\begin{cases}\dfrac{4c_1}{(P_i+P_j)^2} & \text{for}\quad i=1,2,\dots,N,\ \ j=N+1,\dots,2N,\\[2ex] \dfrac{(P_i-P_j)^2}{4c_1} & \text{for}\quad i=1,2,\dots,N,\ \ j=1,2,\dots,N,\\ & \qquad\quad i=N+1,\dots,2N\ \ j=N+1,\dots,2N,\end{cases}$$

and

$$M_1(\mu)=\begin{cases}\exp(\eta_0) & \text{when}\quad 1+\sum_{i=1}^{N}\ \mu_{i+N}=\sum_{i=1}^{N}\ \mu_i\\ 0 & \text{otherwise},\end{cases}$$

$$M_2(\mu)=\begin{cases}\exp(\epsilon_0) & \text{when}\quad 1+\sum_{i=1}^{N}\ \mu_{i+N}=\sum_{i=1}^{N}\ \mu_i\\ 0 & \text{otherwise},\end{cases}$$

$$M_3(\mu)=\begin{cases}1 & \text{when}\quad \sum_{i=1}^{N}\ \mu_{i+N}=\sum_{i=1}^{N}\ \mu_i\\ 0 & \text{otherwise}.\end{cases}\qquad (28)$$

It can be shown by substituting (27) into (6) and then into (3) that the above conclusions are also valid for the higher-order solitons of (q_1,q_2).

IV. DARK SOLITONS

Now in order to construct the exact dark solitons, Eqs. (8) can be decoupled into a different set of bilinear equations for $a=b$ as

$$\mathcal{B}_1 g\cdot f=0,\quad \mathcal{B}_1 h\cdot f=0,\quad \mathcal{B}_2 f\cdot f=-\gamma(gg^*+hh^*),$$
$$(29)$$
$$\mathcal{A}_3 g\cdot h=\mathcal{A}_3 g^*\cdot h=\mathcal{A}_3 g\cdot h^*=\mathcal{A}_3 g\cdot g^*=\mathcal{A}_3 h\cdot h^*=0,$$

where the bilinear operators $\mathcal{B}_1$ and $\mathcal{B}_2$ are defined as

$$\mathcal{B}_1=\left(D_Z-\frac{k'''}{6}\ D_T^3+3\lambda D_T\right),\quad \mathcal{B}_2=\left(\frac{k'''}{6}\ D_T^2-\lambda\right),$$

$$\mathcal{A}_3=D_T,\qquad (30)$$

in which λ is a constant to be determined. Without loss of generality again we choose $a=b=1$ hereafter.

Next for obtaining the dark one-soliton solution, we assume

$$g=\tau_1(1+\chi g_1),\quad h=\tau_2(1+\chi h_1),\quad f=1+\chi f_1,\qquad (31)$$

where τ_1 and τ_2 are complex constants and g_1, h_1, and f_1 are real functions of Z and T. Substituting (31) into (29) and collecting the coefficients of the different powers of χ, we get

$$|\tau_1|^2+|\tau_2|^2=\frac{\lambda}{\gamma} \quad (\chi^0), \tag{32}$$

$$\mathcal{B}_1(1\cdot f_1+g_1\cdot 1)=0, \quad \mathcal{B}_2(1\cdot f_1+h_1\cdot 1)=0,$$

$$\mathcal{B}_2(1\cdot f_1+f_1\cdot 1)+2\gamma(|\tau_1|^2 g_1+|\tau_2|^2 h_1)=0,$$

$$\mathcal{A}_3(1\cdot h_1+g_1\cdot 1)=0 \quad (\chi^1), \tag{33}$$

$$\mathcal{B}_1 g_1\cdot f_1=0, \quad \mathcal{B}_1 h_1\cdot f_1=0,$$

$$\mathcal{B}_2 f_1\cdot f_1+\gamma(|\tau_1|^2 g_1^2+|\tau_2|^2 h_1^2)=0, \quad \mathcal{A}_3 g_1\cdot h_1=0 \quad (\chi^2). \tag{34}$$

One can easily check that the system (32)–(34) admits the following solutions:

$$g_1=h_1=-f_1=-\exp[m_1(T-\lambda Z)+\xi_1^{(0)}], \tag{35}$$

where

$$\lambda=\frac{k'''}{12}m_1^2=\gamma(|\tau_1|^2+|\tau_2|^2). \tag{36}$$

It is obvious from (36) and the condition $k''\gamma=\beta k'''$ that k'' and β have identical sign, which naturally corresponds to the normal GVD region [21] where dark solitons exist. Here m_1 and $\xi_1^{(0)}$ are real constants. Now using (36) in (31) and then in (6), the dark one-soliton solution can be derived as

$$\rho_1=\tau_1\exp(\pm i\pi)\tanh\left\{\frac{1}{2}\left[m_1\left(T-\frac{k'''}{12}m_1^2 Z\right)+\xi_1^{(0)}\right]\right\}, \tag{37a}$$

$$\rho_2=\tau_2\exp(\pm i\pi)\tanh\left\{\frac{1}{2}\left[m_1\left(T-\frac{k'''}{12}m_1^2 Z\right)+\xi_1^{(0)}\right]\right\}, \tag{37b}$$

where the amplitude parameters τ_1 and τ_2 are connected with the ratio k'''/γ by the relation (36) and which is useful for identifying the contribution of k''' and γ in the intensity distribution among (q_1,q_2).

From (37) it is interesting to note that as in the bright soliton case, the velocity of the dark soliton is also proportional to the third-order dispersion and to the square of its width. By using (37) in the transformation (3), one can derive the explicit expressions for (q_1,q_2). Finally, we have not yet been able to obtain dark N-soliton solutions and work is in progress along these lines.

V. EXACT BRIGHT AND DARK SOLITONS WITH THE ADDITIONAL HIGHER-ORDER NONLINEAR TERM

By considering the nonlinear effects resulting from the delayed nonlinear response [22], the coupled NLS system (2) is replaced by

$$iq_{1z}-\frac{k''}{2}q_{1tt}+\beta[(a|q_1|^2+b|q_2|^2)q_1]-\frac{ik'''}{6}q_{1ttt}+i\gamma[(a|q_1|^2+b|q_2|^2)q_1]_t+i\gamma_s(a|q_1|^2+b|q_2|^2)_t q_1=0, \tag{38a}$$

$$iq_{2z}-\frac{k''}{2}q_{2tt}+\beta[(b|q_1|^2+a|q_2|^2)q_2]-\frac{ik'''}{6}q_{2ttt}+i\gamma[(b|q_1|^2+a|q_2|^2)q_2]_t+i\gamma_s(b|q_1|^2+a|q_2|^2)_t q_2=0. \tag{38b}$$

We find that the resulting system (38) still has exact solutions under the earlier conditions, namely, $k''\gamma=k'''\beta$ and $a=b$, provided γ_s is real. So in the following we do not take into account the self-induced Raman effect corresponding to the imaginary part of γ_s [22] but consider only the effect due to the real part. Using the same transformations (3) and (4) in (38), we obtain the following equations under the same earlier condition $k''\gamma=k'''\beta$:

$$\rho_{1Z}-\frac{k'''}{6}\rho_{1TTT}+(\gamma+\gamma_s)(a|\rho_1|^2+b|\rho_2|^2)_T\rho_1+\gamma(a|\rho_1|^2+b|\rho_2|^2)\rho_{1T}=0, \tag{39a}$$

$$\rho_{2Z}-\frac{k'''}{6}\rho_{2TTT}+(\gamma+\gamma_s)(b|\rho_1|^2+a|\rho_2|^2)_T\rho_2+\gamma(b|\rho_1|^2+a|\rho_2|^2)\rho_{2T}=0. \tag{39b}$$

In the absence of γ_s, Eq. (39) reduces to the system (5). Therefore by solving (39) one can find the effect of γ_s on the coupled soliton solutions of the system (2) as derived in Secs. III and IV under the condition $a=b$ and $k''\gamma=k'''\beta$. For this purpose, Eqs. (39) are rewritten using (6) and (7) as

$$f^2\left[\left(D_Z-\frac{k'''}{6}D_T^3\right)g\cdot f\right]+\left[\frac{k'''}{6}D_T^2 f\cdot f+\alpha(agg^*+bhh^*)\right][3D_T g\cdot f]$$
$$+(\gamma+\gamma_s)f[agD_T g^*\cdot g+bhD_T h^*\cdot g+bh^*D_T h\cdot g]=0, \tag{40a}$$

$$f^2\left[\left(D_Z-\frac{k'''}{6}D_T^3\right)h\cdot f\right]+\left[\frac{k'''}{6}D_T^2 f\cdot f+\alpha(bgg^*+ahh^*)\right][3D_T h\cdot f]+(\gamma+\gamma_s)f[ahD_T h^*\cdot h+bgD_T g^*\cdot h+bg^*D_T g\cdot h]=0, \tag{40b}$$

where

$$\alpha=\gamma+\tfrac{2}{3}\gamma_s. \tag{41}$$

As in Secs. III and IV, in order to construct exact bright and dark soliton solutions of Eq. (38), Eq. (40) can be decoupled into two different ways, under the condition $a=b$:

Bright solitons:

$$\left(D_Z-\frac{k'''}{6}D_T^3\right)g\cdot f=0,\quad \left(D_Z-\frac{k'''}{6}D_T^g\right)h\cdot f=0, \tag{42}$$

$$\frac{k'''}{6}D_T^2 f\cdot f+\alpha(gg^*+hh^*)=0, \tag{43}$$

$$D_T g\cdot h=D_T g^*\cdot h=D_T g\cdot h^*=D_T g\cdot g^*=D_T h\cdot h^*=0. \tag{44}$$

Dark solitons:

$$\left(D_Z-\frac{k'''}{6}D_T^3+3\lambda D_T\right)g\cdot f=0,$$

$$\left(D_Z-\frac{k'''}{6}D_T^3+3\lambda D_T\right)h\cdot f=0, \tag{45}$$

$$\frac{k'''}{6}D_T^2 f\cdot f-\lambda f^2=-\alpha(gg^*+hh^*), \tag{46}$$

$$D_T g\cdot h=D_T g^*\cdot h=D_T g\cdot h^*=D_T g\cdot g^*=D_T h\cdot h^*=0. \tag{47}$$

It is interesting to note that the set of Hirota bilinear forms (42)–(44) and (45)–(47) is the same as the previous equations (9) and (29) for the bright and dark solitons, respectively, *except that* α in the latter equations has been replaced by γ. Therefore by making the corresponding replacement in the bright and dark soliton solutions of the system (2) as reported in Secs. III and IV, one can note the following changes in the solutions due to the additional term γ_s.

(i) The coupling equation (20) in the bright soliton case now changes into

$$|\epsilon_1|^2+|\epsilon_2|^2=\frac{-k'''}{3(\gamma+\frac{2}{3}\gamma_s)}. \tag{48}$$

The parametric restriction

$$k''\gamma=k'''\beta \tag{49}$$

is the same whether γ_s is present in (38) or not. Now from (48) and (49), one can argue that either k''' and γ may be allowed to take identical signs [with this restriction, Eq. (49) allows k'' and β to take the same signs only, which naturally corresponds to the normal dispersion region where dark solitons exist [21]] or k''' and γ can be allowed to take opposite signs [with this choice, Eq. (49) restricts the parameters k'' and β to take opposite signs, which naturally corresponds to the anomalous dispersion region where bright solitons exist [19]] by leaving the parameter γ_s to take any value suitable to Eq. (48). It is now clear that bright solitons with some particular values of γ_s can propagate in both the anomalous and normal dispersion regions. The same arguments can also be made for the dark soliton case by using Eq. (49) and the resultant equation obtained from (36) after including the effect of γ_s as

$$|\tau_1|^2+|\tau_2|^2=\frac{k'''m_1^2}{12(\gamma+\frac{2}{3}\gamma_s)}. \tag{50}$$

Thus we note the interesting possibility that both the exact bright and dark solitons of the coupled system (38) can propagate in both the anomalous and normal dispersion regions for some particular choices of γ_s under the conditions $a=b$ and $k''\gamma=k'''\beta$.

(ii) Due to the additional effect of γ_s in (2), the expressions (18) for ϵ_1 and ϵ_2, which appear in the amplitudes of the bright soliton solutions, change into

$$\epsilon_1=\left[\frac{-k'''\ \exp(2\eta_0)}{3(\gamma+\frac{2}{3}\gamma_s)[\exp(\eta_0+\eta_0^*)+\exp(\epsilon_0+\epsilon_0^*)]}\right]^{1/2}, \tag{51a}$$

$$\epsilon_2=\left[\frac{-k'''\ \exp(2\epsilon_0)}{3(\gamma+\frac{2}{3}\gamma_s)[\exp(\eta_0+\eta_0^*)+\exp(\epsilon_0+\epsilon_0^*)]}\right]^{1/2}. \tag{51b}$$

In the case of dark soliton, the amplitude parameters τ_1 and τ_2 are connected by the new equation (50), which reduces to the previous case Eq. (36) if $\gamma_s=0$.

(iii) The phase factor in the bright soliton case also changes into a new form. For example, the Eq. (19) for the phase factor δ_0 in the bright one-soliton solution now becomes, due to the presence of γ_s,

$$\delta_0=\tfrac{1}{2}\ \ln\left[\frac{-3(\gamma+\frac{2}{3}\gamma_s)[\exp(\eta_0+\eta_0^*)+\exp(\epsilon_0+\epsilon_0^*)]}{4k'''P_1^2}\right]. \tag{52}$$

Figure 1 shows the changes in the intensity profile $|q_1|^2+|q_2|^2$ of the bright one-soliton due to the additional effect γ_s, while Fig. 2 shows the same in the absence of γ_s. Similarly, figures can be drawn for the dark soliton solutions.

VI. CONCLUSION

Using the Hirota technique, we have obtained exact bright and dark soliton solutions of the coupled system (2) under the conditions $a=b$ and $k''\gamma=\beta k'''$ so as to get some ideas

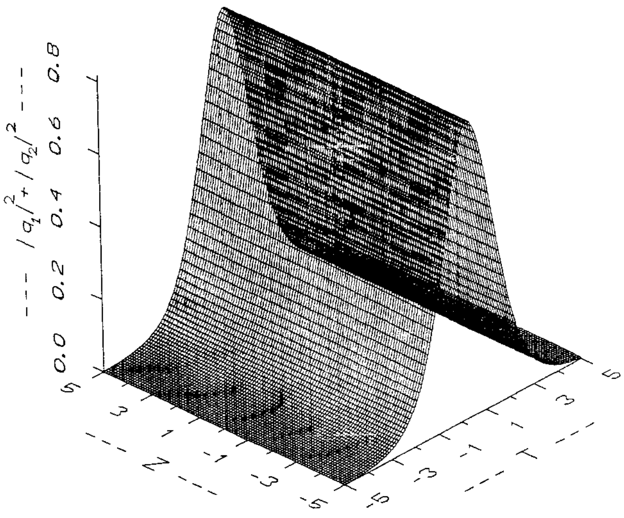

FIG. 1. Intensity profile $|q_1|^2+|q_2|^2$ of the bright one-soliton solution with $\gamma_s=0.006$, while all other parametric values are the same as in Fig. 2. Note the change in amplitude and small shift in the position of the soliton.

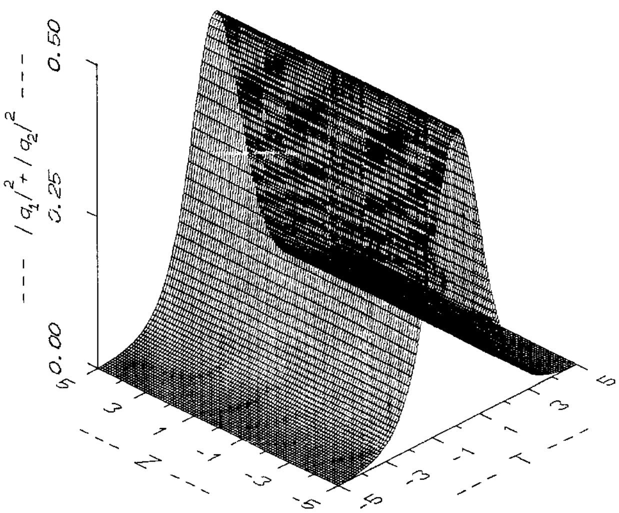

FIG. 2. Intensity profile $|q_1|^2+|q_2|^2$ of the bright one-soliton solution with the parametric choices $k'''=0.03$, $\gamma=-0.01$, $\exp(\eta_0+\eta_0^*)=0.8$, $\exp(\epsilon_0+\epsilon_0^*)=1.2$, and $P_1=0.7$.

about the role of k''' and γ on the coupled soliton dynamics. The results show that the velocity of a soliton is proportional to k''' and to the square of its pulse width. Also we have pointed out that the intensity of a single complex envelope amplitude (say q_1), which is proportional to the ratio k'''/γ in the absence of second envelope, is distributed among (q_1,q_2) while both are present. The dynamics of the bright two-soliton solutions is also studied in detail for confirming its typical behavior. We expect that the simplest form of coupled solitons of the system (2) could be observed experimentally in a properly tailored optical fiber. Finally, we have noted some interesting changes in both the bright and dark soliton solutions of the system (2) when the additional nonlinear term γ_s comes to play.

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